TOOTH CONTACT ANALYSIS OF ZTA TYPE WORM GEAR DRIVES

PÉTER HORÁK, TIBOR BERCSEY
Institute of Machine Design, Budapest University of Technology and Economics
Műegyetem rkp. 3., Budapest, Hungary, H-1111
Horak.Peter@gszi.bme.hu, Bercsey.Tibor@gszi.bme.hu

Abstract: *To calculate the efficiency, to reduce the friction and wear and to optimise the tribology conditions of highly loaded worm gear pairs information is needed about the contact pressure and area between the teeth flanks.*
The simplest theory assuming a constant Hertzian pressure, within a contact position. The aim of this investigation is to turn away from this assumption and to calculate the contact area and the changing load distribution between worm gear teeth pairs through the meshing.
Keywords: *Contact analysis, coefficient matrix, load distribution, worm gear flank.*

1. INTRODUCTION

In the last decades, due to the new theories and manufacturing processes of gear pairs, significant development occurred in the field of worm gear pairs, widely applied to the transmission of motion and power. Theoretical and experimental investigations have proved that, significant improvement in load-carrying capacity and efficiency can only be achieved if hydrodynamic lubrication and the corresponding contact conditions furthermore an improvement of manufacturing precision are realised.

To calculate the efficiency, to reduce the friction and wear and to optimise the tribology conditions of highly loaded worm gear pairs information is needed about the contact pressure and area between the teeth flanks.

The worm gear teeth surfaces contacting theoretical along a line. The radii of curvature are continuously changed depending on the meshing position along the contact line. The simplest theory assuming a constant Hertzian pressure, within a contact position, along the instantaneously contact line.

The aim of this investigation is to turn away from this assumption and to calculate the contact area and the changing load distribution between worm gear teeth pairs through the meshing. Figure 1 shows the main steps of the numerical investigation.

The results of this investigation can be used to calculate the tribology condition between the tooth surfaces.

The subject of this investigation is a worm gear pair having circular arch profile in axial section. This type of worm drive (ZTA) was developed at the University of Miskolc and manufactured at the Machine Factory in Diósgyőr. Several papers by Professor Dudás and his research team at the Department of Production Technology of the University Miskolc relate about the manufacturing process, geometry and meshing of ZTA type worm gears [1], [3], [4].

In a close cooperation with the Institute of Machine Design of the Budapest University of Technology and Economics the contact and the tribology conditions was investigated [5]. This paper may contribute to further cooperation between our institutes on the field of the investigation of gear drives.

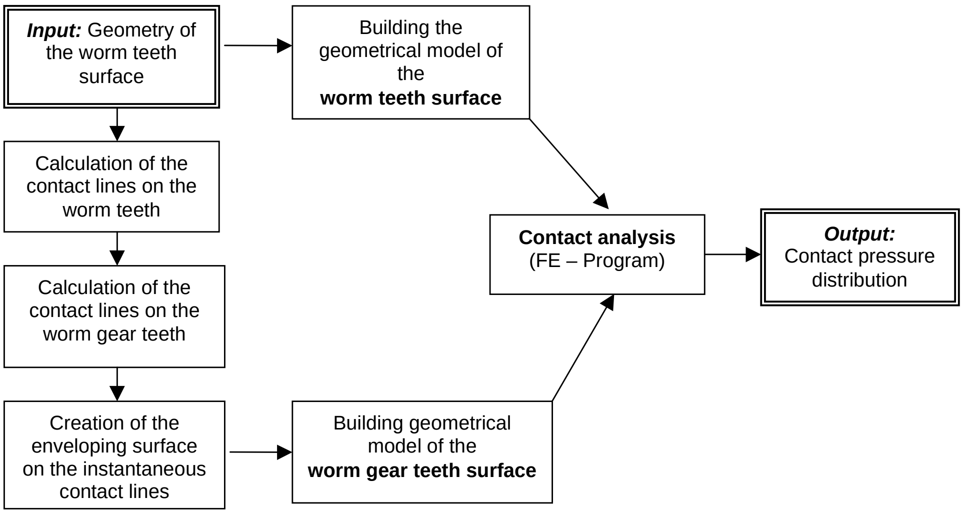

Fig. 1. The main steps of the calculation

2. GEOMETRY MODEL OF WORM AND WORM GEAR

The geometry model was built by means of the parametric equations of the worm [3] in the CAD System ProEngineer.

The main data of the investigated worm gear pairs are as follows:

Number of teeth: $z_1=3$

Modul: m=12.5 mm

Centre distance: a=280 mm

Profile angle: α=23°03’40’’

Radius of the arch: ρ=50 mm

The instantaneous contact lines were calculated by means of the kinematics method [6], through the meshing (Fig. 2).

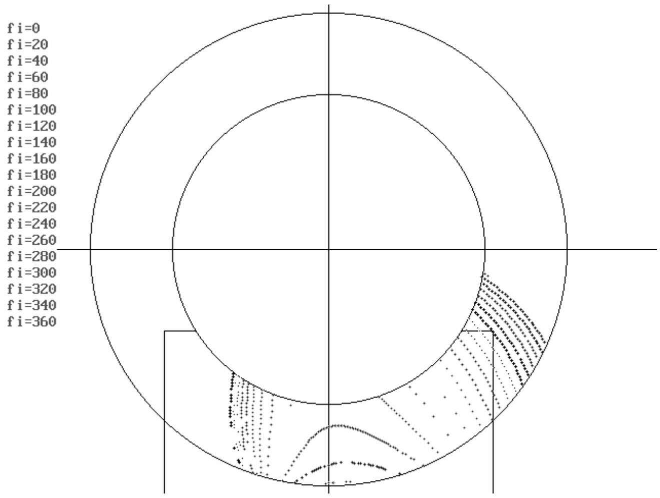

Fig. 2. Contact lines on the worm flank

The model of worm gear flank was built as enveloping surface of the contact lines in the coordinate system of the worm gear.

Another way to build the geometry model is the method of axial sections, published in [2].

Using the built in FEM modul of ProEngineer, automatic mesh generation by means of TETRA elements having 4 nodes was applied.

3. CONTACT ANALYSIS

There are different methods for the calculation of the load distribution and contact area. Some modern Finite Element Analysis software (for example MSC Marc) is able to solve contact problems without using of special contact elements.

For the numerical solution we have used an algorithm based upon the coefficient matrix [7].

Figure 3 represent the geometrical condition of a simple 2D point contact, where δ_i the rigid-body displacement is in accordance with the sum of the initial gap h_i and the elastic displacements u_i, in all points of the contact area.

$$\delta_i = u_i^1 + h_i^1 + u_i^2 + h_i^2 \quad (i = 1,...N) \qquad (1)$$

The equation of the contact stress in the inner points of contact is:

$$p_i > 0, \quad (1 < i < N), \qquad (2)$$

while in the initial and terminal points it is:

$$p_1 \equiv 0 \quad \text{and} \quad p_N \equiv 0. \qquad (3)$$

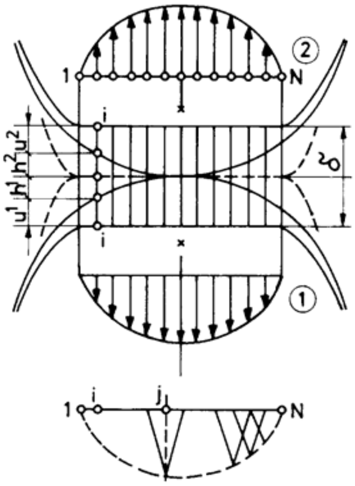

Fig. 3. Geometric conditions of contact

Elastic displacement u_i of the contact points can be derived by means of the finite element method. Elastic displacement arises by the effect of pressure distribution p_j. According to Cauchy's superposition theorem w_{ij} denote the displacement of nodal point i caused by the triangular unit pressure contained in j. With knowledge of the effect of each pressure distribution, it can be written for the elastic body No. 1 as follows:

$$u_i^1 = \sum_{j=1}^{N} w_{ij}^1 p_j \qquad (i = 1,...N) \qquad (4)$$

The elastic displacement of elastic body No. 2 u_i^2 can be derived on the basis of w_{ij}^2.

The geometric equation of the contact is:

$$\delta_i = \sum_{j=1}^{N} \left(w_{ij}^1 + w_{ij}^2 \right) p_j + h_i^1 + h_i^2 \qquad (i = 1,...N). \qquad (5)$$

Let the sum of the coefficient matrix of the elastic bodies be designed by symbol w_{ij}, while its inverted by symbol $(w_{ij})^{-1}$.

Rearranging equation (5) the equation (6) gives the contact pressure distribution.

$$p_j = \sum_{i=1}^{N} \left(w_{ji} \right)^{-1} \left(\delta_i - h_i^1 - h_i^2 \right) \qquad (6)$$

In case of worm gear flanks line contact occurs theoretical instead of point contact, so the initial gaps h_i are equal to zero and the length of the contact area is also given.

The coefficient matrix can be generated during a single run by the finite element program suitable for taking more load cases into consideration.

Figure 4 shows the calculated contact pressure along the contact lines on the flanks of worm and worm gear for a selected meshing position.

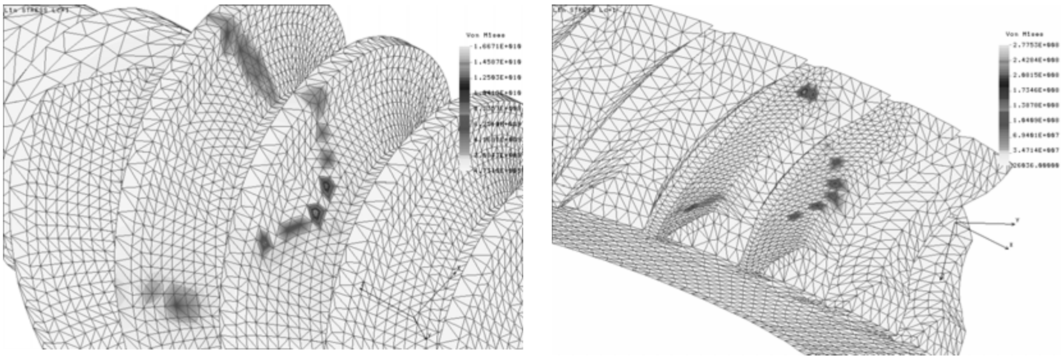

Fig. 4. Calculated contact pressure distribution on worm and worm gear flanks

The results of the calculations building the input data of further calculations relating the tribology conditions and optimisation of the gear pairs. Investigating of different meshing position a significant change of the maximal Hertzian pressure can be observed (Fig. 5).

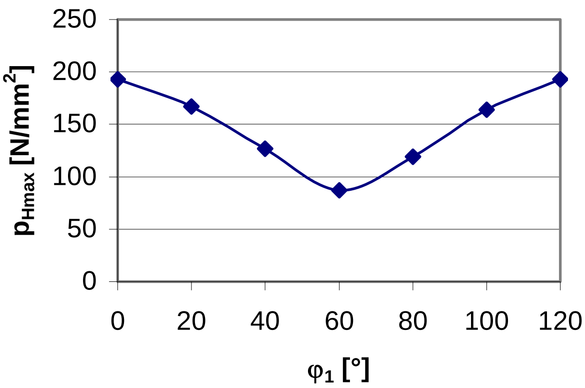

Fig. 5. Changing maximal contact pressure at different meshing position

Investigating of different geometry we can state, increasing of the profile shift of the worm gear, the average contact pressure increases along the contact lines during the meshing. Figure 6. shows the influence of profile shift of the investigated worm gear on the average Hertzian pressure.

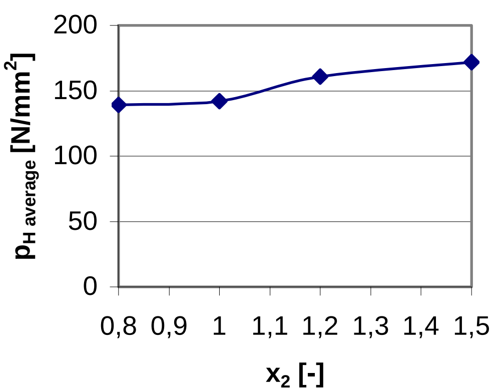

Fig. 5. Average contact pressure as function of profile shift of the worm gear

4. CONCLUDING REMARKS

A computer program has been developed to calculate the instantaneously contact lines by means of the kinematics method on cylindrical worm teeth surfaces. The enveloping surface of the instantaneously contact lines gives the worm gear tooth surface. The generated worm and worm gear bodies are the input data for the numerical solution of the contact problem.

Finite element analysis has been used for the calculation of the displacement, the load distribution along the contact lines. Based on the results, we can say that the maximum stresses are caused by the Hertzian contact of the tooth surfaces on the contact line and are changing depending on the meshing position. The influence of deflection of the worm shaft on the meshing and bearing pattern and is not significant. Parameter studies have proved, the profile shift of the worm gear influences significantly of the average contact pressure along the instantaneous contact lines during the meshing.

The results of the investigation can be used to calculate the friction condition between the tooth surfaces.

ACKNOWLEDGEMENT

The presented research was supported by the OTKA T 034 968 research program.

5. REFERENCES

1. Ankli, J.; Drobni, J.; Dudás, I.; Garamvölgyi, T.: Berendezés és eljárás főmetszetben ívelt profilú csigahjtópár geometriailag helyes gyártására alkalmas köszörűkorong profilírozására. 170 118 sz. szabadalmi leírás, megjelent 1987. január 17.
2. Dierich, H.: Weiterentwicklung der Theorie zur Ermittlung von Hertzschen Drücken und Reibungszahlen in Verzahnungen von Schneckengetrieben. Dissertation Ruhr-Universität Bochum, 1989.
3. Dudás, I.: The Theory and Practice of Worm Gear Drives. Penton Press, London, 2000. (ISBN 1 8571 8027 5)
4. Dudás, I.; Bányai, K.; Varga, Gy.: Simulation of meshing of worm gearing. ASME 7th International Power Transmission and Gearing Conference, San Diego, 1996. Proceedings, pp. 141-146.
5. Horák, P.: Computer model of the contact relations of worm gear pairs. 4th World Congress on Gearing and Power Transmission, Paris, 16-18 March, 1999. pp. 483-488.
6. Litvin, F. L.: Gear geometry and applied theory. Englewood Cliffs, Prentice Hall, NJ., 1994
7. Váradi, K.; Poller, R.: Analysis of gear teeth contact by the finite element method. Acta Technica Acad. Sci. Hung., 101(4), pp. 397-416, 1988.