VIBRATORY STALK-CUTTING EXCITED BY VARIANCE OF MATERIAL STREAM

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Abstract: A specially designed and developed coupling equipment (patent name: MULTIKÁTOR) applied in driving cutting mechanism (ie on forage harvesters) excites the motion in the direction of cutting-edge and results energy saving. Such a system could be modeled as a univariant oscillating system. The applied torque on the oscillating system can be divided into two parts: on the one hand a deterministic applied and on the other hand a stochastic loading torque originating from the cutting process. For this reason the excitation of the oscillating system is a stochastic signal. We present the theoretical method to evaluate the model with help of the random variable of the excitation signal we will characterize the probability parameters of angular speed and oscillation of the MULTIKÁTOR. The method can be adopted for similar model as well.

Key words: stochastic analyses, machine design, process engineering, cutting

1. INTRODUCTION

Mechanical cutting processes have special importance in some areas of engineering (ie. agricultural and industrial material processing). Energy consumption of the process can be optimized with applying auxiliary edgewise motion of knives. At Szent Istvan University a special driving system – called “Multikator” – had been developed and published [1, 3, 5] for further utilization at different agricultural machinery- especially on forage harvesters (Fig.1).

Fig. 1. Prototype of the Multikator and the schema of flexible coupling

During chopping processed material provides stochastic excitation causing periodic oscillation of the system. The objective of this paper is to investigate the character of the behaviour of the given flexible driving system. As the basis of theoretical investigation real-time tests had been carried out identifying operational parameters (ie. angular velocity and
acceleration, torque etc). The model has been developed according to the registered real parameters.

2. DESCRIPTION OF THE STOCHASTIC PROCESSES

The exciting of the examined vibrating system, for their function, is stochastic, since the incipient forces are also stochastic. For this reason the characterization of the dynamical processes can be possible by the description methods of the stochastic signals. The mentioned methods are [4]:

a) Assembly of the realization of stochastic signals,
b) Assembly of the probability density functions,
c) Moments of the probability density functions (mean value, correlation functions, etc.),
d) Special features, for example junction points of given signal level, peaks, enveloping curve, maximum and minimum point, time of the first given signal level junction point, etc.

The remarkable stochastic processes are: Gaussian normal, Poisson- and the Markov-processes. Great advantage of the normal processes is: it is fully characterisable by the quadratic moments. Since the statistical quantities relating to the linear operations are closed. It is also a common knowledge that according to the central limit theorem the considerable amount of the natural processes are normal. The Poisson processes (and the other processes derived from that) can be featured by only one parameter. The Markov processes are rather universal, since they have few approaches relating to the specific processes, for this reason they are not widely spread.

3. THE STOCHASTIC PROCESS CONTROL

The exciter functions

The oscillating system is linear, thus the principle of superposition is applicable for that. Therefore it has a sense for dividing the resultant torque exciting signal to components. According to the evaluation the exciting torque can be divided 3 components: constant, harmonic and stochastic component. The amplitude of the stochastic signal is rather considerable, approximately 3 times the standard deviation (≈ 200 – 300 Nm), for that the stochastic signal analysis should be executed also. The stochastic signal is a normal distribution noise, for this reason the analysis of the correlation functions is applicable [2, 5].

For the examinations the autocorrelation function of torque, angular frequency and angular acceleration should be used up. Two of the signals found in the decomposition are deterministic and one of them is stochastic. The response functions for the deterministic torque signal can be determined with the use of common routine. This paper is dealing with the noise component only.

Assume that the \( x_m(t) \) stochastic torque signal is ergodic, so its autocorrelation function can be determined form only one infinite time domain representation [2, 4]:

\[
\phi_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x_m(t) x_m(t + \tau)
\]

In case of \( \tau = 0 \) zero translation the autocorrelation function gives the mean-square value of the signal (effective value), which is the square of the standard deviation for stochastic signal [5].
\[
\varphi_{mn}(0) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x_m(t)x_n(t) = \sigma^2. \quad (G2)
\]


\[
\Phi_{mn}(j\omega) := F[\varphi_{mn}(\tau)] = X_m(j\omega)X^*(j\omega),
\]

\[
X_m(j\omega) := F[x_m(t)]. \quad (G3)
\]

The \( F \) means the Fourier transform and the asterisk the complex conjugate in the G3, respectively. The mentioned equation has just a theoretical importance, but it will be shown to be very useful for formal calculations.

**Response functions for flexible pulley**

The response function of angular rotation, angular frequency and the angular acceleration will be examined. The connection between the exciting and response functions is the system motion equation:

\[
\Theta \frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + Dx(t) = M(t) \quad (V1)
\]

It can be write in a slightly modified way:

\[
\frac{d^2x(t)}{dt^2} + 2\zeta \omega_0 \frac{dx(t)}{dt} + \omega_0^2 x(t) = m(t), \quad (V2)
\]

Where \( x(t) \) is the angular rotation in radians, \( \zeta \) is the attenuation coefficient, \( \omega_0 \) is the undamped angular natural frequency, \( m(t) \) is the resultant torque signal referring to the inertial moment. From the viewpoint of examinations the stationer signals have high importance, for this reason these will be calculated. The angular rotation for the time constant \( M_e \) torque is:

\[
x_e := \frac{M_e}{\Theta \omega_0^2} \quad (V3)
\]

from the earlier mentioned equations.

The harmonic excitation torque signal with \( \omega_g \) angular frequency and \( \hat{M}_h \) amplitude has a

\[
\hat{X} = \frac{\hat{M}_h}{\sqrt{(\omega_0^2 - \omega_g^2)^2 + 4\zeta \omega_0 \omega_g^2}} \quad (V4)
\]

motion equation and

\[
\beta := \text{Arg} \frac{-2\zeta \omega_0 \omega_g}{\omega_0^2 - \omega_g^2} \quad (V5)
\]

phase displacement of harmonic angular oscillation.

Let’s see the characterization of angular oscillation caused by the stochastic torque.

It is a matter of course that just the statistical parameters are computable such as the \( \sigma_X \) standard deviation of angular oscillation. For the determination of standard deviation of angular oscillation let’s take the (V2) equation, substitute the exciting torque, take the Fourier transform of that and put the deflection angle. In this way:
Having proved that the stochastic exciting torque is a Gaussian band-limited white noise, the autocorrelation function of the angular frequency is countable from the equation:

\[
\Phi_{XX}(j\omega) = X(j\omega)X^*(j\omega) = \frac{m_{\omega \omega}(j\omega)m_{\omega \omega}^*(j\omega)}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} = \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \Phi_{\omega \omega \omega}(j\omega),
\]

(V7)

It is easy to understand the autocorrelation function of the angular frequency is countable from the equations:

\[
\Phi_{XX}(j\omega) = X(j\omega)X^*(j\omega) = \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \Phi_{\omega \omega \omega}(j\omega),
\]

(V8)

and the angular acceleration from the equations:

\[
\Phi_{XX}(j\omega) = X(j\omega)X^*(j\omega) = \frac{\omega^4}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \Phi_{\omega \omega \omega}(j\omega).
\]

(V9)

Using the inverse Fourier transform the relevant autocorrelation function can be computed from the above mentioned equations. According to the (G2) the standard deviation is computable with the \( \tau = 0 \) substitution

\[
\sigma_X^2 = \int_{-\infty}^{\infty} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \Phi_{\omega \omega \omega}(j\omega) d\omega,
\]

\[
\sigma_X^2 = \int_{-\infty}^{\infty} \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \Phi_{\omega \omega \omega}(j\omega) d\omega,
\]

(V10)

\[
\sigma_X^2 = \int_{-\infty}^{\infty} \frac{\omega^4}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \Phi_{\omega \omega \omega}(j\omega) d\omega.
\]

where the point above the parameters means the derivation with the respect of time. The signal processing has proved that the stochastic exciting torque is a Gaussian band-limited white noise. The autocorrelation function of that is:

\[
\Phi_{\omega \omega \omega}(j\omega) = \frac{1}{\Theta^2} \frac{\sigma_{\omega \omega}^2}{2\Delta\omega},
\]

(V11)

where the \( \Delta\omega \) is the band-width (approximately \( \approx 150 \frac{1}{s} \)). Having substituted that into the (V10) equation and employed the even argument:

\[
\sigma_X^2 = \frac{1}{\Theta^2} \frac{\sigma_{\omega \omega}^2}{\Delta\omega} \int_{0}^{\Delta\omega} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} d\omega,
\]

\[
\sigma_X^2 = \frac{1}{\Theta^2} \frac{\sigma_{\omega \omega}^2}{\Delta\omega} \int_{0}^{\Delta\omega} \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} d\omega,
\]

(V12)

\[
\sigma_X^2 = \frac{1}{\Theta^2} \frac{\sigma_{\omega \omega}^2}{\Delta\omega} \int_{0}^{\Delta\omega} \frac{\omega^4}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} d\omega.
\]
Unfortunately the integrals above can not be given in closed formation; therefore an approximation should be applied. In the present case the band with is much wider then the resonance frequency of the system, in this case the upper limit of the integral can be infinite. Thus the integrals are countable:

\[
\sigma_x^2 = \frac{1}{\Theta^2} \frac{\pi}{4 \zeta \omega_0^2} \frac{\sigma_{m1}^2}{\Delta \omega},
\]

\[
\sigma_x^2 = \frac{1}{\Theta^2} \frac{\pi}{4 \zeta \omega_0^2} \frac{\sigma_{m2}^2}{\Delta \omega},
\]

\[
\sigma_x^2 = \frac{1}{\Theta^2} \frac{\pi}{4 \zeta \omega_0^2} \frac{\sigma_{m3}^2}{\Delta \omega}.
\]

(V13)

It can be seen from the above mentioned equations if the \( \zeta \) attenuation factor converges to zero, the response functions will converge to infinity. Thus the stochastic resonance appears.

**Level intersection**

Determine the frequency of plane intersection to explain the stochastic resonance. Sign the \( N^+(\alpha)dt \) the probability of the situation when the \( X(t) \) stochastic signal crosses the \( \alpha \) signal level with positive slope in the \([t, t + dt]\) interval.

This probability is:

\[
N_x^+(\alpha)dt = P\left[ X(t) < \alpha \cap \left\{ X dt \geq \alpha - X(t), \dot{X}(t) > 0 \right\} \right] = \\
= \int_{a-\omega dt}^{a} \int_{0}^{x} p(\dot{x}, x)dx dt = \int_{0}^{\infty} \int_{0}^{x} p(\dot{x}, x)dx \Rightarrow \\
\Rightarrow N_x^+(\alpha) = \int_{0}^{\infty} x p(\dot{x}, x)dx.
\]

(S1)

where the \( p(\dot{x}, x) \) is the density function of joint distribution and the \( N^+(\alpha) \) is the frequency of upward intersection. The frequency of the downward intersection, which is the frequency of the intersection of negative slope, can be countable in like manner:

\[
N_x^-(\alpha) = - \int_{-\infty}^{0} x p(\dot{x}, x)dx.
\]

(S2)

The signals are Gaussian in the examined situation thus the density function of joint distribution is

\[
p(\dot{x}, x) = \frac{1}{2\pi \sigma_x \sigma_{\dot{x}}} e^{-\frac{x^2}{2\sigma_x^2} - \frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}}.
\]

(S3)

With the help of (SZ1) (SZ2) (SZ3) equations the frequency of the level intersections are.

\[
N_x^+(\alpha) = N_x^-(\alpha) = \frac{\sigma_x}{2\pi \sigma_x} e^{-\frac{\alpha^2}{2\sigma_x^2}}.
\]

(S4)

If the \( \alpha = 0 \) it is the so called zero point. The frequency of that according to the above mentioned equation is:

\[
N_x^+(0) = N_x^-(0) = \frac{\sigma_x}{2\pi \sigma_x}.
\]

(S5)
Apply the solution to flexible pulley. According to the (V13):

\[ N_X^+(0) = N_X^-(0) = \frac{a_0}{2\pi} = f_0, \quad (S6) \]

where \( f_0 \) is the oscillating frequency of the undamped system.

**According to our solutions** the reason of the stochastic resonance is the coincidence of zero point frequency of the stochastic exciting torque and the oscillation frequency of the undamped system

**The time of the first level intersection and the maximal value of signals**

The time of the first level intersection is the \( T_f \) period whereafter the \( X(t) \) stochastic signal intersect the \( \alpha \) level at the first time. If the level is high enough then it can be assumed that the individual level intersections are independent so it is fulfill the requirements of Poisson distribution. Therefore the probability of the number of the level intersections (n) in the \( [0,t] \) period is:

\[ P(n,t) = \frac{(f t)^n}{n!} e^{-\beta}. \quad (K1) \]

The probability of the absence of the junctions in the mentioned interval is:

\[ P_{T_f}(t) = P[T_f > t] = P(0,t) = e^{-\beta}. \quad (K2) \]

The probability of at least 2 junctions in the mentioned interval is:

\[ F_{T_f}(t) = P[T_f \leq t] = 1 - P(0,t) = 1 - e^{-\beta}. \quad (K3) \]

Hence the probability density of the first level intersection can be determined by the differentiation of that.

\[ p_{T_f} = \frac{dF_{T_f}(t)}{dt} = fe^{-\beta}. \quad (K4) \]

The standard deviation and the expected value of the first junction is:

\[ \langle T_f \rangle = \int_0^\infty t p_{T_f} = f^{-1}, \quad \sigma_{T_f}^2 = \int_0^\infty (t - \langle T_f \rangle)^2 p_{T_f} = f^{-2} \quad (K5) \]

Therefrom it is apparent that:

\[ f = N_X^+(\alpha). \quad (K6) \]

The probability of \( X_m < \alpha \) is:

\[ P_{X_m}(\alpha) = P[X_m < \alpha] = P[T_f > t] = P_{T_f}(T) \quad (K7) \]

where \( X(t) \) is the stochastic signal, \( X_m \) is the maximum of stochastic signal in \( [0,t_0 + T] \) interval.

So it is connection with the probability distribution function on the first level intersection. Hence the probability density function and the expected value and standard deviation of the maximum value is determinable. So the solution of the development is [2]:

\[ \langle X_m \rangle = C \sigma_X, \quad \sigma_{X_m} = \frac{\pi}{\sqrt{6}} \frac{\sigma_X}{C_1}. \quad (K8) \]

where

\[ C = C_1 + \frac{0.5772}{C_1}, \quad \text{and} \quad C_1 = \sqrt{2\ln(N_X^+(0)T)} \quad (K9) \]

are the computational coefficients.
The above mentioned notions have been acting as amplitude in stochastic processes. It can be seen that the expected value and the standard deviation of the maximum value are the function of the period for examination.
In case of increasing period of duty the standard deviation of maximal value is decreasing, the expected value is increasing with infinitely increasing series. If the maximum value exceeds the permissible value in viewpoint of stress limits or quality, malfunction will happen. If the tolerance level (e.g.: limit deviation) is known, then the time of the first malfunction is determinable.

4. REFERENCES