STRESS ANALYSES OF CYLINDRICAL VESSEL WITH CHANGEABLE HEAD GEOMETRY

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Abstract: The main objective of this paper is numerical analyses of cylindrical pressure vessel with changeable head geometry (semi-elliptical and hemispherical heads) and comparison of results in means of precision and time needed for getting the solution. Manufacture technology of pressure vessel cylinders and heads are described, and construction geometry of various pressure vessel heads is presented. Discretisation style of finite element models, boundary conditions, and results are detailed shown and explained. Based on analytical solutions for membrane stress state of pressure vessel, numerical analysis of it is performed also. Comparison of harvested numerical results for above mentioned pressure vessels are shown tabular, with values of time needed for getting the solution too. And comparison of analytical and numerical results for pressure vessel with hemispherical heads is shown too.

Key words: cylindrical pressure vessel, hemispherical head, semi-elliptical head, finite element method, ANSYS.

1. INTRODUCTION

Pressure vessels are inevitably elements of thermal systems, hydro systems, chemic system, water supply systems etc. It is presumed that pressure vessel is every tank with volume of \( V \geq 1.5 \text{ m}^3 \), inner pressure bigger than 0.05 MPa, or negative pressure smaller than 0.08 MPa, and accumulated energy \( p \cdot V \geq 0.02 \text{ MPa} \cdot \text{m}^3 \).

2. MANUFACTURE TECHNOLOGY OF PRESSURE VESSEL

Parts of cylindrical pressure vessels are manufacturing separately, cylinder and heads. Cylinder is made from segments which are welded in one unit, and heads are joining with cylinder by welding too, as shown on Fig. 1.

Fig. 1 Welded cylindrical pressure vessel [1]
Manufacture of cylinder is beginning with preparing sheet metal in means of cutting on right dimensions and preparation of edges which must be welded longitudinal. Afterwards, before final bending, sheet metal must be pre-bended in purpose of correct longitudinal joint (after final bending) for welding, what is condition for good quality of welded joint. After welding the cylinder, it must be calibrated to eliminate possible oval shape of cylinder. Then, header edges for joint of heads must be prepared for welding too. Heads are manufacturing with procedure of segment pressuring. Manufacture is consisting from following operations: prepare of material, segment pressuring (Fig. 2), getting final form of head (Fig. 3), preparing edge for welding (Fig. 4). After that, head is finished and prepared for welding to cylinder.

Figures from Fig. 5 to Fig. 8 are showing different types of pressure vessel heads.

3. MEMBRANE SHELL THEORY

Shells can be thin-walled and thick-walled. Thin-walled shells are restricted with ratio of plate thickness and inner or outer radius of shell, as shown in following equations:

\[
\frac{s}{R_i} \leq \frac{1}{20}, \quad \frac{s}{R_o} \leq \frac{1}{20}
\]

(1)

Shells with larger value of ratio from equations (1) are thick-walled shells. Fig. 9 shows stress distribution per plate thickness. That distribution is per hyperbolic state for thick-walled shells (Fig. 9, a), and per linear state for thin-walled shells (Fig. 9, b).
3.1 PRESSURE VESSEL WITH HEMISPHERICAL HEADS

For calculating Von Mises equivalent stress, hoop and longitudinal stresses are needed, which have to be calculated from the hoop and longitudinal forces.

a) Pressure vessel cylinder

- Hoop force:
  \[ N_h = \frac{p \cdot R_p}{s}, \]  \( (2) \)
  where \( p \) is inner pressure and \( R_m \) is average radius of curvature:
  \[ R_m = R_i + \frac{R_o - R_i}{2}, \]  \( (3) \)

- Longitudinal force:
  \[ N_l = \frac{p \cdot R_o}{2}, \]  \( (4) \)

- Hoop stress:
  \[ \sigma_h = \frac{N_h}{s}, \]  \( (5) \)

- Longitudinal stress:
  \[ \sigma_l = \frac{N_l}{s}, \]  \( (6) \)

After hoop and longitudinal stresses are calculated, it is possible to calculate Von Mises equivalent stress [5]:

\[ \sigma_{eq} = \sqrt{\sigma_h^2 + \sigma_l^2 - \sigma_h \cdot \sigma_l}. \]  \( (7) \)

b) Pressure vessel head

In case of pressure vessel head, hoop and longitudinal force are the same:

\[ N_h = N_l = \frac{p \cdot R_o}{s}, \]  \( (8) \)
Hoop and longitudinal stresses are same too, and because of that, maximal Von Mises stress of pressure vessel head is equal to hoop and longitudinal stresses [5]:

\[ \sigma_h = \sigma_1 = \frac{N_h}{s} = \frac{N_1}{s}, \quad \sigma_{eq} = \sigma_h = \sigma_1. \]  

(9)

c) Radial displacements

-Pressure vessel cylinder

\[ \Delta R_{cylinder} = \frac{p \cdot R^2_m}{2 \cdot E \cdot s_{cylinder}} \cdot (2 - \nu) \]  

(10)

-Pressure vessel head

\[ \Delta R_{head} = \frac{p \cdot R^2_m}{2 \cdot E \cdot s_{head}} \cdot (1 - \nu) \]  

(11)

d) Condition for membrane stress state

For acquiring a membrane stress state in pressure vessel, plate thickness of cylinder must be greater than plate thickness of head. Adversely, on pressure vessel joint of cylinder and head, apart from membrane stresses, bending stress will appear too, so plate thickness of cylinder must be greater than plate thickness of head, according to following equation [5].

\[ s_{cylinder} = \frac{2 - \nu}{1 - \nu} \cdot s_{head}. \]  

(12)

4. GEOMETRY OF CONSIDERED PRESSURE VESSELS

Pressure vessels considered in this paper are completely the same, only difference is in type of heads. There are two cases: pressure vessel with hemispherical heads (Fig. 10), and pressure vessel with semi-elliptical heads (Fig. 11). In both cases, internal pressure is \( p = 1.6 \) MPa, inner diameter is \( d_i = 5170 \) mm, cylinder length of pressure vessel is \( L = 8000 \) mm and plate thickness is \( s = 30 \) mm.
5. ANALYTICAL CALCULATION OF PRESSURE VESSEL WITH HEMISPHERICAL HEADS

Analytical calculation is performed for pressure vessel with hemispherical heads. Dimensions and internal pressure are shown on Fig. 10. In this calculation, only Von Mises equivalent stress for pressure vessel cylinder is calculated, because from membrane shell theory (chapter 3.1) it is obvious that maximal Von Mises equivalent stress will appear on pressure vessel cylinder, and not on head. Radial displacements of pressure vessel cylinder and head, and condition for acquiring a membrane stress state in pressure vessel are calculated too, because of afterward numerical analysis. Results are following:

\[ \sigma_{eq} = 120,089 \text{ MPa}; \quad \Delta R_{cylinder} = 1,4593 \text{ mm}; \quad \Delta R_{head} = 0,6008 \text{ mm}; \quad s_{cylinder} = 72,857 \text{ mm} \]

6. NUMERICAL ANALYSIS WITH FINITE ELEMENT METHOD

Analysis of pressure vessels in this paper is realised with software ANSYS 11. Three types of finite elements are used: SOLID 95, PLANE 183 and SHELL 181, which are described in table 1. Material is set as isotropic, linear elastic with Young's modulus of elasticity \( E = 210000 \) MPa, and Poisson's ratio \( \nu = 0,3 \).

Because of total symmetry of pressure vessel - geometry and load, only 1/8 of pressure vessel is modeled, and because of that, numerical analysis is much simpler and time for getting solution is much shorter.

<table>
<thead>
<tr>
<th>SOLID 95</th>
<th>Description of element</th>
</tr>
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<tbody>
<tr>
<td>SOLID 95</td>
<td>SOLID 95 is defined by 20 nodes having three degrees of freedom per node: translations in the nodal ( x ), ( y ), and ( z ) directions. The element may have any spatial orientation. SOLID95 has plasticity, creep, stress stiffening, large deflection, and large strain capabilities. It can tolerate irregular shapes without as much loss of accuracy. SOLID 95 elements have compatible displacement shapes and are well suited to model curved boundaries.</td>
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</tbody>
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<table>
<thead>
<tr>
<th>PLANE 183</th>
<th>Description of element</th>
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</thead>
<tbody>
<tr>
<td>PLANE 183</td>
<td>PLANE 183 is defined by 8 nodes or 6 nodes having two degrees of freedom at each node: translations in the nodal ( x ) and ( y ) directions. The element may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an axisymmetric element. This element has plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It is a higher order 2-D, 8-node or 6-node element. PLANE 183 has quadratic displacement behavior and is well suited to modeling irregular.</td>
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<tr>
<th>SHELL 181</th>
<th>Description of element</th>
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<tbody>
<tr>
<td>SHELL 181</td>
<td>SHELL181 is suitable for analyzing thin to moderately-thick...</td>
</tr>
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</table>
shell structures. It is a 4-node element with six degrees of freedom at each node: translations in the $x$, $y$, and $z$ directions, and rotations about the $x$, $y$, and $z$-axes. (If the membrane option is used, the element has translational degrees of freedom only). SHELL 181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses.

6.1 NUMERICAL ANALYSIS OF PRESSURE VESSEL WITH HEMISPHERICAL HEADS

Fig. 12 and Fig. 13 are showing finite elements mesh and boundary conditions for pressure vessel with hemispherical heads. Because of limited length of this paper, only finite elements mesh for element SOLID 95 is shown (Fig. 12). For all types of finite elements, mesh is generated automatically, except on joint of cylinder and head, where mesh is set up with more density. Fig. 13 is showing boundary conditions. Red arrows are representing inner pressure of 1.6 MPa, and blue symbols are representing proposed symmetry of pressure vessel. Because of totally symmetry of pressure vessel, symmetry is set up on all three edges of pressure vessel part.

Fig. 14 is showing results for Von Mises equivalent stress for pressure vessel with hemispherical heads meshed with finite element SOLID 95. As expected, the maximal equivalent stress is appearing on pressure vessel cylinder with amount of 122,997 MPa for inner pressure of $p = 1.6$ MPa.

Fig. 12 Finite elements mesh for pressure vessel with hemispherical heads

Fig. 13 Boundary conditions for pressure vessel with hemispherical heads
Fig. 14 *Von Mises equivalent stress for inner pressure $p=1.6$ MPa - hemispherical heads*

Fig. 15 is showing parallel comparison of equivalent stress results for all three type of finite elements. It is presented only detail of pressure vessel joint of head and cylinder. As figure shows, stress distribution for all three types of elements is almost identical. In cases of elements *PLANE 183* and *SOLID 95* it is possible to see stress distribution per plate thickness of pressure vessel, but in case of element *SHELL 181* it is not possible, and that is disadvantage in using of this element for numerical analysis. Fig. 15, c) is showing stress distribution on inner wall of pressure vessel.

Fig. 15 *Parallel comparison of equivalent stress results - hemispherical heads; a) PLANE 183, b) SOLID 95, c) SHELL 181*
6.1.1 NUMERICAL ANALYSIS OF MEMBRANE STRESS STATE

For acquiring a membrane stress state in pressure vessel, plate thickness of cylinder must be greater than plate thickness of head. Analytical calculation is performed and it is obtained that plate thickness of cylinder must be 72,857 mm. With numerical analysis it is ascertain that with increase thickness of cylindrical plate from 30 to 72,857 mm, there is still a bending. It is obvious from this example that is impossible to acquire membrane stress state, because membrane stress state neither don’t exist, because bending will appear always, in bigger or smaller amount.

From Fig. 16 it is obvious that equivalent stress values around joint of cylinder and head are almost the same with amount of \( \approx 68.5 \, \text{MPa} \). Away from joint of cylinder and head, according to cylinder, stress values are decreasing for about \( \approx 20 \, \text{MPa} \) and becomes constant in continues. Away from joint, according to head vertex, stresses are constant. So, membrane stress state is prevailed in most of pressure vessel cylinder and head, but not in joint of cylinder and head.

Fig. 16 Stress distribution (Von Mises) through plate thickness for pressure vessel with hemispherical heads, and with inner pressure of \( p = 1.6 \, \text{MPa} \)

Fig. 17 Radial displacement for pressure vessel with hemispherical heads, and with inner pressure of \( p = 1.6 \, \text{MPa} \)

Fig. 17 is showing radial displacement regarding to axis of symmetry of pressure vessel. As figure shows, radial displacements around joint of cylinder and head are almost the same with value of \( \approx 0.56 \, \text{mm} \). Away from joint of cylinder and head, according to cylinder, displacement values is decreasing for very small amount, and becomes constant in continues. Away from joint, according to head vertex, displacement values is approaching value 0. That is logic, because in axis of symmetry of pressure vessel, there is no radial displacement.

6.2 NUMERICAL ANALYSIS OF PRESSURE VESSEL WITH SEMI-ELLIPtical HEADS

In this case of pressure vessel with semi-elliptical heads will be shown only finite elements mesh and boundary conditions for element SHELL 181, because for elements PLANE 183 and SOLID 95 it is same as in case of hemispherical heads, only difference is of course in geometry of heads. Fig. 18 and Fig. 19 are showing finite elements mesh and boundary conditions for pressure vessel with semi-elliptical heads. As in case of pressure vessel with hemispherical heads, in all cases of finite elements, mesh is generated automatically, expect on joint of cylinder and head, where mesh is set up with more density. In this case of head, mesh is denser on head vertex too, because of small hole which should be created there.
Fig. 19 is showing boundary conditions. Red arrows are representing inner pressure of 1.6 MPa, and blue symbols are representing propose symmetry of pressure vessel. As apposed in earlier case of hemispherical heads, in this case there are orange symbols too, which represents constraint of rotation. That constraint must be set, because element \textit{SHELL 181} has rotational degree of freedom around certain axis too.

Reason for the small hole placing at the head vertex lies in fact that singularity of stress appears at this place. Because of singularity, value of stress on head vertex is varying with number of finite elements, during values of stresses in all rest pressure vessel is the same. So, to avoid this appearance, small hole on head vertex should be created, and on edge of that hole, constraint of displacement in radial direction must be set. This constrain is justified to set, because created hole is assume as endless small, like a point on axis of symmetry of pressure vessel, and on axis of symmetry, there is no radial displacement. In such of set boundary conditions, singularity of stresses on head vertex is avoided, and stress distribution of pressure vessel is correct.

![Finite elements mesh for pressure vessel with semi-elliptical heads](image1)

**Fig. 18** Finite elements mesh for pressure vessel with semi-elliptical heads

![Boundary conditions for pressure vessel with semi-elliptical heads](image2)

**Fig. 19** Boundary conditions for pressure vessel with semi-elliptical heads

Fig. 20 is showing the distribution of equivalent stress for pressure vessel with semi-elliptical heads meshed with finite element \textit{SHELL 181}. As expected, the maximal equivalent stress is appearing on minimal meridian curvature of pressure vessel head with amount of 206.081 MPa for inner pressure of \( p = 1.6 \) MPa.

![Equivalent stress field for inner pressure \( p = 1.6 \) MPa - semi-elliptical heads](image3)

**Fig. 20** Equivalent stress field for inner pressure \( p = 1.6 \) MPa - semi-elliptical heads
Fig. 21 is showing parallel comparison of equivalent stress results for all three types of finite elements. It is presented only detail of minimal meridian curvature of pressure vessel head, because that is the place of maximal value of stress. As figure shows, stress distribution for all three types of elements is almost identical. Fig. 21, c) is showing stress distribution on inner surface wall of pressure vessel.

![Fig. 21 Parallel comparison of equivalent stress results-semi-elliptical heads; a) PLANE 183, b) SOLID 95, c) SHELL 181](image)

7. COMPARISON OF NUMERICAL RESULTS OBTAINED BY USING OF DIFFERENT TYPES OF ELEMENTS FOR BOTH HEADS GEOMETRY

Tables 2 and 3 are showing comparison of numerical results for both cases of pressure vessel heads and for all three types of elements. Table 2 is showing comparison of results for pressure vessel with hemispherical heads, and table 3 is for pressure vessel with semi-elliptical heads.

**Table 2. Comparison of numerical results for the pressure vessel with hemispherical heads**

<table>
<thead>
<tr>
<th>Element type</th>
<th>Maximal equivalent stress $\sigma_{eq}$, MPa</th>
<th>Mesh parameters, calculating time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLID 95</td>
<td>122,997</td>
<td>Elements: 136 908, Nodes: 217 542, Time: 9 min, 31 s</td>
</tr>
<tr>
<td>PLANE 183</td>
<td>122,867</td>
<td>Elements: 3 214, Nodes: 7 361, Time: 2 s</td>
</tr>
<tr>
<td>SHELL 181</td>
<td>124,051</td>
<td>Elements: 20 294, Nodes: 10 316, Time: 10 8 s</td>
</tr>
</tbody>
</table>
### Table 3. Comparison of numerical results for the pressure vessel with semi-elliptical heads

<table>
<thead>
<tr>
<th>Element type</th>
<th>Maximal equivalent stress $\sigma_{eq}$, MPa</th>
<th>Mesh parameters, calculating time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLID 95</td>
<td>209,438</td>
<td>Elements: 145 320</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nodes: 229 084</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time: 9 min, 27 s</td>
</tr>
<tr>
<td>PLANE 183</td>
<td>209,073</td>
<td>Elements: 5 364</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nodes: 11 727</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time: 2,7 s</td>
</tr>
<tr>
<td>SHELL 181</td>
<td>206,081</td>
<td>Elements: 26 186</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nodes: 13 344</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time: 12,9 s</td>
</tr>
</tbody>
</table>

As tables show, results for maximal equivalent stresses for all three types of elements and in both cases of head type are almost identical. Numerical analysis with finite elements SOLID 95 requires much more elements, because it is need to mesh a volume with plate thickness of 30 mm. And because of bigger number of elements, time for calculating is much longer too. In case of element SHELL 181 it is needed to mesh a shell, so number of elements and time for calculating is much smaller. And in case of element PLANE 183 it is need to mesh only a plane, and in this case number of elements and time for calculating is the smallest of course.

Table 4 is showing comparison of numerical and analytical results for pressure vessel with hemispherical heads, and percent discrepancy of results. For numerical results is taken only those calculated with element PLANE 183, and results for analytical calculating is given in chapter 5.

### Table 4. Comparison of numerical and analytical results for pressure vessel with hemispherical heads, percent discrepancy

<table>
<thead>
<tr>
<th>Type of calculating</th>
<th>Maximal equivalent stress $\sigma_{eq}$, MPa</th>
<th>Radial displacements $\Delta R$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cylinder ($\Delta R_{cylinder}$)</td>
</tr>
<tr>
<td>Analytical solution</td>
<td>120,089</td>
<td>1,4593</td>
</tr>
<tr>
<td>Numerical result</td>
<td>122,867</td>
<td>1,488</td>
</tr>
<tr>
<td>Percent discrepancy, %</td>
<td>2,26</td>
<td>1,92</td>
</tr>
</tbody>
</table>

As table shows, all results discrepancy is in allowed boundary of 5%, what means that results are acceptable!

### 8. CONCLUSION

In this paper a numerical analysis of pressure vessel with hemispherical and semi-elliptical heads is performed, with three types of elements: SOLID 95, PLANE 183 and SHELL 181. It is concluded that in both cases of pressure vessel heads, using of PLANE 183 element...
presents the best approach, because of minimal number of elements for meshing, shortest calculation time, insight into the stress distribution per plate thickness and obtained results which are closest to the analytical ones. This type of axisymmetric element could be recommended in such cases, when the total symmetry of model is considered (geometry and load).

Analysis of cylindrical pressure vessel with different head types is performed in purpose of comparison of values of maximal equivalent stresses. It is concluded that smaller values of equivalent stresses are appearing in pressure vessel with hemispherical heads, and equivalent stress distribution is advantageous too in that case of head geometry. Nevertheless, in exploitation, there is much more pressure vessels with semi-elliptical heads, because manufacturing of hemispherical heads is much more complicated and expensive. Maximal equivalent stress for considered vessel geometry in case of hemispherical heads is appear in pressure vessel cylinder wall with amount of 123 MPa, and in case of semi-elliptical heads, maximal equivalent stress is almost double (209 MPa) and it is appear on minimal meridian curvature of pressure vessel head, what is expecting from shell of revolution theory. Again, nevertheless pressure vessels with semi-elliptical heads are much more manufacturing, because of earlier mention reason.

It is analytically calculated that for acquiring a membrane stress state in considered pressure vessel, plate thickness of cylinder must be \( h_{\text{cylinder}} = 72.857 \) mm. Afterwards numerical analysis is approved that completely membrane stress state in vessel is impossible to acquire, because bending will appear always, in bigger or smaller amount.

9. REFERENCES


