MODELING OF ELASTO-PLASTIC MATERIALS IN FINITE ELEMENT METHOD

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Abstract: User-defined material models which can be used with finite elements commercial programs make possible to define any constitutive model of arbitrary complexity. Developer should be concerned with the expansion of the material model without the development and maintenance of FEM software. Modeling of several elastic-plastic material models is presented in this paper. Classical theory of plasticity is used. Two types of integration routines are used: explicit and implicit. Additionally unified plasticity Bodner-Partom material model is presented. It takes into consideration such nonlinear effects as plasticity, isotropic and kinematic hardening, visco-plasticity and creep.

Key words: plastic flow theory, user-defined material, Bodner-Partom material.

1. INTRODUCTION

Material models available in professional finite element programs although effective, sometimes are not sufficient to model sophisticated nonlinear processes. User-defined materials may be helpful in such cases. ABAQUS – one of the best FEM programs available on the market – requires appropriate procedure written in FORTRAN [1] to define material constitutive equations. This procedure called UMAT [2] is compiled and linked to solver. The large number of input parameters enables careful analysis of the current solution state and allows to determine the next solution increment. In plasticity problems various yield conditions, hardening rules and hardening parameters may be used [3]. Rate dependent plasticity and creep are major concerns in the structural analysis in materials beyond the yield limit. The Bodner-Partom viscoplastic material model with creep [4] is an example of modern sophisticated constitutive equations modeling. A major difficulty of practical B-P model applications is the large number of material parameters. Although the methodology of experimental evaluation of these parameters is known, such measurements are more complex than those in the classical plasticity or viscoplasticity.

In this paper presented is the elastic-plastic material model with isotropic hardening rule. The classical plastic flow theory is used. Explicit and implicit integration schemes are implemented. Engineering applications of elastic-plastic material model are shown. Unified plasticity Bodner-Partom material model is also presented. Results for one dimensional tension test are already obtained. The numerical procedure for 3D case is still under development.
2. ELASTIC-PLASTIC ISOTROPIC HARDENING MATERIAL MODEL

In this paper the following symbols are used: \( \sigma \) - stress tensor, \( \sigma' \) - deviatoric stress tensor, \( \varepsilon \) - strain tensor, \( \mathbf{I} \) - second order identity tensor, \( \sigma_e \) - effective stress, \( \sigma_Y \) - yield stress, \( \varepsilon_p \) - effective plastic strain. The expression \( \sigma : \varepsilon \) denotes double contraction of tensors i.e. \( \sigma : \varepsilon = \sigma_{ij} \varepsilon_{ij} \). Effective stress and effective plastic strain are defined as:

\[
\sigma_e = \sqrt{\frac{3}{2} \sigma' : \sigma'}
\]

\[
\varepsilon_p = \sqrt{\frac{2}{3} \varepsilon''' : \varepsilon'''}
\]  

In the paper the following conditions are valid.

Decomposition of strains into elastic and plastic parts:

\( \varepsilon = \varepsilon^e + \varepsilon^p \)  

(3)

Incompressibility condition:

\( \mathbf{I} : \varepsilon''' = \varepsilon_{11}'' + \varepsilon_{22}'' + \varepsilon_{33}''' = 0 \)  

(4)

Von Mises yield criterion (\( \mathbf{f} \ll \mathbf{0} \) elastic deformation, \( \mathbf{f} = \mathbf{0} \) plastic deformation):

\( \mathbf{f} = \sigma_e - \sigma_Y = \sqrt{\frac{3}{2} \sigma' : \sigma' - \sigma_Y} \)  

(5)

Normality condition (flow theory):

\( d\varepsilon^p = d\lambda \frac{\partial \mathbf{f}}{\partial \sigma} \)  

(6)

Consistency condition:

\( df = \frac{\partial \mathbf{f}}{\partial \sigma} d\sigma + \frac{\partial \mathbf{f}}{\partial \varepsilon_p} d\varepsilon_p = 0 \)  

(7)

In the case of von Mises material model:

\( d\lambda = d\varepsilon_p = \sqrt{\frac{2}{3} \varepsilon''' : \varepsilon''' \varepsilon'''} \)  

(8)

Based on the above equations appropriate numerical procedure is developed. Two types of integration routines are used: forward Euler (explicit) and backward Euler (implicit) [7]. Schematic representation of both integrations is presented in Fig. 1.
Explicit integration methods are simple but have a stability limit i.e. $\Delta t < \Delta t_{stabl}$. For explicit integration the time increment must be controlled [5]. On the other hand, implicit integration is unconditionally stable, but the numerical procedure is complicated. To avoid the updated stresses taking outside the yield surface, at first the trial stress is chosen (elastic predictor) and then the stress is updated to bring it back onto the yield surface (plastic corrector).

## 2. BODNER-PARTOM MATERIAL MODEL

The Bodner-Partom material model originally created with only isotropic hardening has been modified to incorporate kinematic hardening, viscoplasticity, creep and recovery in various temperatures. Bodner-Partom material does not require an explicit yield condition (yield surface). However, an appropriate choice of material parameters allows to approximate a sharp yield point. A major difficulty of practical B-P model applications is the large number of material parameters. Although the methodology of experimental evaluation of these parameters is known, such measurements are more complex than those in the classical plasticity or viscoplasticity. The general form of B-P theory is formally analogous to von Mises plasticity. It is worth mentioning that the plastic multiplier in the flow law is not found from the consistency condition, but is obtained on the basis of micro-mechanical level investigations.

The strain tensor consists of elastic and inelastic parts

$$\varepsilon = \varepsilon^{(E)} + \varepsilon^{(I)}$$  \hspace{1cm} (9)

The elastic stress components are provided by the generalized Hooke’s law

$$\sigma^{(E)} = D \varepsilon^{(E)} = D (\varepsilon - \varepsilon^{(I)})$$  \hspace{1cm} (10)

where $D$ is the elastic stiffness tensor.

The equation for inelastic strain rate of the B-P formulation is given by
The quantities $D_0$, $Z_0$, $Z_1$, $Z_2$, $Z_3$, $m_1$, $m_2$, $n$, $A_1$, $A_2$, $r_1$, $r_2$ are material parameters (see table 1.).

\[ \hat{\mathbf{b}}^{(ie)} = \frac{\sqrt{3}D_0}{\alpha^0} \exp \left[ -\frac{1}{2} \left( \frac{Z}{\sigma} \right)^{2n} \right] \sigma' \]

(11)

It is consistent with Prandtl-Reuss flow law. The coefficient $D_0$ is the limiting strain rate in shear for large stress. The parameter $n$ controls the rate sensitivity. The hardening parameter $Z$ represents the resistance to inelastic deformation and consists of isotropic component $Z^i$ and limiting value for kinematic hardening $Z^0$ value of the directional hardening tensor $\mathbf{b}$. The quantities $Z^i$ and $Z^0$ are load history dependent and are obtained by the following evolution equations:

\[ \dot{Z}^i = m_1 (Z_1 - Z^i) \sigma_i \hat{\mathbf{b}}^{(ie)} - A_1 Z_1 \left( \frac{Z^i - Z_2}{Z_1} \right)^n \]

(12)

\[ \dot{Z}^0 = m_2 \left( Z_3 \frac{\sigma}{\|\sigma\|} - \mathbf{b} \right) \sigma_i \hat{\mathbf{b}}^{(ie)} - A_2 Z_1 \left( \frac{\|\mathbf{b}\|}{Z_1} \right)^n \]

(13)

where $\|\sigma\| = \sqrt{\sigma_i \sigma}$, $m_1, m_2, Z_1, Z_2, Z_3, n_1, n_2$ are material parameters (see table 1.).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0$</td>
<td>s$^{-1}$</td>
<td>Limiting shear-strain rate</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Pa</td>
<td>Initial value of isotropic hardening variable</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>Pa</td>
<td>Limiting value for isotropic hardening</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>Pa</td>
<td>Fully recovered value for isotropic hardening</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>Pa</td>
<td>Limiting value for kinematic hardening</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Pa$^{-1}$</td>
<td>Hardening rate coefficient for isotropic hardening</td>
</tr>
<tr>
<td>$m_2$</td>
<td>Pa$^{-1}$</td>
<td>Hardening rate coefficient for kinematic hardening</td>
</tr>
<tr>
<td>$n$</td>
<td>-</td>
<td>Strain rate sensitivity parameter</td>
</tr>
<tr>
<td>$A_1$</td>
<td>s$^{-1}$</td>
<td>Recovery coefficient for isotropic hardening</td>
</tr>
<tr>
<td>$A_2$</td>
<td>s$^{-1}$</td>
<td>Recovery coefficient for kinematic hardening</td>
</tr>
<tr>
<td>$r_1$</td>
<td>-</td>
<td>Recovery exponent for isotropic hardening</td>
</tr>
<tr>
<td>$r_2$</td>
<td>-</td>
<td>Recovery exponent for isotropic hardening</td>
</tr>
</tbody>
</table>

3. ENGINEERING APPLICATIONS

In this chapter presented are some engineering applications of user-defined elastic-plastic material model. As the first one the sheet rolling problem is considered. The problem is highly...
non-linear because of contact, large displacements and non-linear constitutive equations [6]. The load increments should be carefully defined in order to obtain the convergence in Newton-Raphson iterations. The remeshing procedure might be necessary. Sheet rolling numerical analysis shows the usefulness of developed user-defined material in solving sophisticated highly non-linear material processing problems [8].

![Fig. 2. Sheet rolling problem](image)

The simulation of forging is the next problem solved in this research. A well lubricated rigid die moves down to deform a blank of rectangular cross-section. The problem is assumed to be axisymmetric, large displacements are chosen. The indentation depth is about 50% of the original blank thickness. The die is modeled with an analytical rigid surface.

![Fig. 3. Effective plastic strain in die forging simulation](image)
4. CONCLUSION

An implementation of user-defined materials in ABAQUS program requires good knowledge of: theory of finite element method, ABAQUS user interface, programming techniques and theory of incremental plasticity. Writing non-linear material procedures is an ambitious and laborious task. On the other hand, user-made numerical procedures may consider various parameters and additional features not analyzed by commercial FEM programs. Moreover, an alternative approaches may be implemented here. A good example are fuzzy logic and neural network algorithms applied to modeling of metal forming by researches in recent years. Several problems of theory of plasticity (some of them are not mentioned in this paper) are solved by use of standard ABAQUS materials and user-defined material. The conformities of obtained results are very promising. Sophisticated material models combined with complex loads require use of the most advanced material implementations. Classical theory of plasticity leads to very complicated formulations. Bodner-Partom material model which is the representative of unified plastic theories allows for obtaining reasonable results in such case in a relatively easy way.

6. REFERENCES