The Study of Gearing Line’ Limits of the Cycloid Gear With Roller Teeth

Dăscălescu, Anamaria

Abstract: From dynamic point of view, at the cycloid gear transmission is not rational to utilize the entire length of the cycloid profile of the satellite gear. The favorable interval of the cycloid profile of a tooth is limited by the tooth addendum’ circle with \( r_a \) radius and the tooth dedendum’ circle with \( r_b \) radius. The dynamic favorable interval represent \( \approx 45\% \) from entire cycloid profile length of a satellite gear tooth.

Keywords: gearing line, pressure angle, reaction values.

1 INTRODUCTION

The study of the gearing line’ limits of the cycloid gear with roller teeth includes determinations on two types of cycloid reducer with roller teeth: M1C-23 with gear ratio \( i_1 = z_3 = 17 \), number of roller teeth \( z_2 = 18 \), eccentricity \( a = 2.4 \) [mm], driving engine power \( P = 1.1 \) [kW], the driving engine’ nominal speed \( n_m = 1500 \) [rot/min] and the roller tooth diameter \( d_{so} = 10 \) [mm], respectively RPR – 150 with gear ratio \( i_1 = z_2 = 25 \), number of roller teeth, \( z_2 = 26 \), eccentricity \( a = 2 \) [mm], driving engine power \( P = 1.5 \) [kW], the driving engine’ nominal speed \( n_m = 1500 \) [rot/min] and the roller tooth diameter \( d_{so} = 12 \) [mm].

The pressure angle between a roller tooth and the cycloid profile represents an essential indicator on the estimation process of the energetic flux transmission (forces). The satellite gear’ cycloid profile optimization takes into account the variation of this angle during a cycle of gearing of a roller tooth. When the pressure angle is zero, or has small values, the energetic flux transmission is carry out in good condition.

Based on the transmission angle \( \gamma(\varphi) \) variations’ graphs, we conclude that the central interval of the cycloid profile of a tooth, between \( 13^\circ - 119^\circ \), is favorable to the transmission of the forces and the beginning/ending part of the cycloid profile of a tooth corresponds to some transmission angle unacceptable for the forces.

We conclude also that the entire gear line utilization, from \( \varphi = 0^\circ \) .. \( 180^\circ \), is not possible because we see the pressure angle \( \gamma \) tends to \( 90^\circ \) on the extremity parts of the gearing line, which proves that exists blocking situations.

The analyze of the \( R_{12}, R_{13}, R_{23} \) reactions’ variation graph shows us that, theoretically, the reactions are infinite at the beginning/ending of the gearing of the sun gear roller tooth with the satellite gear cycloid profile. The reactions decrease in the central part of the active profile and, practical, became constant on the interval \( \varphi = 20^\circ - 120^\circ \).

By dynamical point of view, on the forces transmission is not rational to use the entire length of the cycloid profile. The cycloid profile’ favorable interval to transmit the forces is delimited by the tooth addendum circle with \( r_a \) radius and the tooth dedendum circle with \( r_b \) radius. In the paper, we present the mathematical expressions of the tooth addendum circle radius, respectively tooth dedendum circle radius. These circles limit the optimal cycloid profile considering the forces transmission favorable angle.

The dynamic favorable interval represent \( \approx 45\% \) from entire cycloid profile length of a satellite gear tooth. We obtain this shorten length of the gearing line by supplementary manufacturing process of the beginning / ending part of the tooth’ active profile. To use only the dynamic favorable domain of the cycloid profile is possible by elimination of the beginning / ending part of the tooth’ active profile through decreasing the tooth addendum circle’ diameter, respectively through deepening the tooth dedendum profile.

2 THE PRESSURE ANGLE AND THE GEARING LINE

At the cycloid gear, the pressure angle \( \gamma \) of the roller tooth and the cycloid profile is formed between the normal direction of the CA segment in the gearing point of a roller tooth and the line AD, perpendicular to the centre line direction (fig.1), (1). When the pressure angle is zero, or has small values, the energetic flux transmission is carry out in good condition.

\[
\gamma_i = \arctan \left( \frac{r_c \cos \varphi_i - r_a}{r_b \sin \varphi_i} \right)
\]

(1)

In the case of the cycloid gear with X-profile, the gearing line don’t pass through the gearing pole (extrapolette gearing), (fig. 1).

We consider the beginning of the gearing line in the contact point of the roller tooth with the tooth dedendum of the satellite gear. Is situated at \( \rho_{min}=r_a-r_b-r_{bo} \), distance from the pole C.

The ending of the gearing line is in the contact point of the roller tooth with the tooth addendum of the satellite gear. Is situated at \( \rho_{max}=r_a+r_a-r_{bo} \), distance from the pole C.

The gearing line’s points are situated on the normal line of the cycloid profile, in the contact points of the roller tooth with the cycloid profile, at \( r_{so} \), distance from the point B of the theoretical profile.

In the coordinate system fixated on the sun gear with roller teeth, the parametric equations of the gearing line points’ coordinate are:

\[
x_{Ai} = r_g + (\rho_{bi} - r_{bo}) \sin \gamma_i
\]

(2)

\[
y_{Ai} = (\rho_{bi} - r_{bo}) \cos \gamma_i
\]

(3)

Based on the determination relations of the reactions' values, we present the variation of the $R_{12}(\phi)$, $R_{32}(\phi)$, and $R_{42}(\phi)$ reactions who act on the satellite gear, calculated in MathCAD utilitarian program, for different position of the satellite gear expressed by the rotation angle of the centroids tangent point, $\phi$. (fig. 2).

$$R_{12}(\phi) = \frac{10M_1}{2a} \left( \frac{F_1}{\cos(\phi)} - \frac{F_2}{\sin(\phi)} \right)$$

$$R_{32}(\phi) = \frac{10M_1}{2a} \left( \frac{F_3}{\cos(\phi)} - \frac{F_4}{\sin(\phi)} \right)$$

$$R_{42}(\phi) = \frac{10M_1}{2a} \left( \frac{F_5}{\cos(\phi)} - \frac{F_6}{\sin(\phi)} \right)$$

$\phi = 0..180$

![Fig. 2. The $R_{12}(\phi)$, $R_{32}(\phi)$, and $R_{42}(\phi)$ reactions graphs who act on the cycloid satellite gear - cicloyd reducer M1C-23](image.png)
Fig. 3. The variation of the pressure angle corresponding to M1C-23

\[ \gamma(\phi) = \frac{\pi}{180} - \frac{rg}{2} \sqrt{\frac{ex^2}{2} + \frac{1}{2} - \frac{2ex \cos(\gamma(\phi))}{ex \sin(\gamma(\phi))}} - \frac{rbo}{\pi} \quad \gamma(\phi) = a \sin \left( \frac{r3 \cos(\gamma(\phi)) - rg}{\rho(\gamma(\phi)) + rbo} \right) \quad \phi = 0 \ldots 180 \]

\[ \text{root}(\gamma(\phi), \phi) = 52.145 \]
\[ \text{root}(\gamma(\phi) - 50, \phi) = 16.768 \]
\[ \text{root}(\gamma(\phi) + 50, \phi) = 116.768 \]

Fig. 4. The \( R_{32}(\phi) \), \( R_{23}(\phi) \) și \( R_{42}(\phi) \) reactions graph who act on the cycloid satellite gear - cycloid reducer RPR – 150

\[ \phi = \begin{cases} 18 \quad 7140 \quad 3121 \quad 8679 \\ 36 \quad 4792 \quad 2488 \quad 4582 \\ 54 \quad 4416 \quad 2473 \quad 3354 \\ 72 \quad 4982 \quad 2658 \quad 2886 \\ 90 \quad 5110 \quad 3003 \quad 2751 \\ 108 \quad 6104 \quad 3588 \quad 3011 \\ 126 \quad 7903 \quad 4627 \quad 3693 \\ 144 \quad 10206 \quad 6753 \quad 5531 \\ 162 \quad 27174 \quad 33933 \quad 13948 \end{cases} \]

\[ \phi = \begin{cases} 18 \quad 7140 \quad 3121 \quad 8679 \\ 36 \quad 4792 \quad 2488 \quad 4582 \\ 54 \quad 4416 \quad 2473 \quad 3354 \\ 72 \quad 4982 \quad 2658 \quad 2886 \\ 90 \quad 5110 \quad 3003 \quad 2751 \\ 108 \quad 6104 \quad 3588 \quad 3011 \\ 126 \quad 7903 \quad 4627 \quad 3693 \\ 144 \quad 10206 \quad 6753 \quad 5531 \\ 162 \quad 27174 \quad 33933 \quad 13948 \end{cases} \]

Fig. 5. The variation of the pressure angle corresponding to RPR – 150

\[ \gamma(\phi) = \frac{\pi}{180} - \frac{rg}{2} \sqrt{\frac{ex^2}{2} + \frac{1}{2} - \frac{2ex \cos(\gamma(\phi))}{ex \sin(\gamma(\phi))}} - \frac{rbo}{\pi} \quad \gamma(\phi) = a \sin \left( \frac{r3 \cos(\gamma(\phi)) - rg}{\rho(\gamma(\phi)) + rbo} \right) \quad \phi = 0 \ldots 180 \]

\[ \text{root}(\gamma(\phi), \phi) = 44.971 \]
\[ \text{root}(\gamma(\phi) - 50, \phi) = 12.551 \]
\[ \text{root}(\gamma(\phi) + 50, \phi) = 112.551 \]
We present the graphs of the \( R_{12x}, R_{12z}, R_{42z} \) reactions’ variations, figures 2 and 4, calculate in hypothesis I (each satellite gear gears with a roller tooth and a thumb -of the transversally coupling- on the entire length of the gearing line). Analyzing the reactions graphs, we conclude that, in the beginning/ending of the roller tooth gearing with the cycloid profile, theoretical the reactions are infinite and decrease on the central interval of the active profile, practical being constant on the interval \( \varphi=20^\circ-120^\circ \).

The accepted values of the transmission angle we consider to be until 50°, and these accepted values are situated between the limits 12,951° and 112,951° of the angle \( \varphi \) for the first analyzed reducer, respectively 16,768° and 116,768° of the angle \( \varphi \) for the second analyzed reducer.

Analyzing the graphs of transmission angle variations \( \gamma(\varphi) \) (fig. 3 and 5), we consider that the central interval of the cycloid profile limited by 13°-119° is favorable to the forces transmission. The beginning / ending of the cycloid profile correspond to some transmission angles unacceptable for the forces. The usage of the entire length of the gearing line, from \( \varphi=0^\circ \) ... 180°, is not rational, because, at the ending part of the gearing line, the pressure angle increase to 90° (fig. 3 and 5) showing us the existing possibilities of blocking.

We conclude that, from dynamic point of view, to transmit the forces is not rational to utilize the entire length of the cycloid profile.

From point of view of the transmission of the forces, the favorable interval of the cycloid profile is limited by the tooth addendum circle with \( r_a \) radius and the tooth dedendum circle with \( r_f \) radius.

We present the determination relation of these circles’ radius values (4.) Portiunile profilului cicloidal avanta joase din punct de vedere al transmisiunii forțelor sunt delimitate de cercul de cap de raza \( r_a \) și cercul de picior \( r_f \).

\[
r_{a,f} = \sqrt{x_{A,f}^2 + y_{A,f}^2}
\]

where: \( x_{A,a} \) and \( y_{A,a} \), respectively \( x_{A,f} \) and \( y_{A,f} \), are the coordinate of the gearing entrance point, respectively the coordinate of the gearing exit point of a tooth.

\[
x_{A,f} = r_a - a + \left( \sqrt{r_g^2 + r_g^2 - 2r_g r_a \cos \varphi_{a,f}} - r_{a0} \right) \frac{r_a \cos \varphi_{a,f} - r_a}{\sqrt{r_g^2 + r_g^2 - 2r_g r_a \cos \varphi_{a,f}}}
\]

\[
y_{A,f} = \left( \sqrt{r_g^2 + r_g^2 - 2r_g r_a \cos \varphi_{a,f}} - r_{a0} \right) \frac{r_f \sin \varphi_{a,f}}{\sqrt{r_g^2 + r_g^2 - 2r_g r_a \cos \varphi_{a,f}}}
\]

In figure 6 we present, with thick line, the effectively length of the gearing line between the limits \( \varphi \) and \( \varphi_1+\varphi_2 \). The active interval of the cycloid profile corresponds to a-b interval to the direction of rotation, respectively c-d to the inverse direction of rotation.

The dynamic favorable interval represent 45% from entire cycloid profile length of a satellite gear tooth.

We obtain this shorten length of the gearing line by supplementary manufacturing process of the beginning / ending part of the tooth’ active profile. To use only the dynamic favorable domain of the cycloid profile is possible by elimination of the beginning / ending part of the tooth’ active profile through decreasing the tooth addendum circle’ diameter corresponding to b-c interval, respectively through deepening the tooth dedendum profile corresponding to d-e interval.

The cycloid reducer has multiple advantages that are not found in any other type of reducer: the reducing report is until 119:1 in one step, the impute pour is between 30W and 200KW, the gear transmission efficiency on a reducing level is until 98% because the rolling of the elements, a robust design, the shafts alignment and perfectly symmetrical position of all pieces ensure a smooth and silence function with very low level of vibration, the centrifuge force are compensate by using two satellites gear dispose at 180° or three satellite gear dispose at 120°. The disposal of the shafts on the same axe allows a direct transmission of the power and fast assembly.

The most important disadvantage of the cycloid gear reducer is the complex form of the cycloid gear profile which involves the necessity of high precision manufacturing. A high precision manufacturing technology implies precision machines tools and expensive tools. Also, an important disadvantage is that the modification of axial distance modifies the instantaneous transmission report.

All the advantages, plus the low gauge and mass to the same power and the same transmission report as in a traditional gear reducers and the extended period of working hours without repairing in heavy work conditions, recommends this type of gear reducer.
4. RESULTS AND CONCLUSIONS

Analyzing the graphs of $R_{12}$, $R_{32}$, $R_{42}$ reactions’ variations, figures 2 and 4, calculate in hypothesis I (each satellite gear gears with a roller tooth and a thumb -of the transversally coupling- on the entire length of the gearing line), we conclude that, in the beginning/ending of the roller tooth gearing with the cycloid profile, the theoretical reactions are infinite and decrease on the central interval of the active profile, practical being constant on the interval $\varphi=20^\circ-120^\circ$.

Analyzing the graphs of transmission angle variations $\gamma(\varphi)$, we consider that the central interval of the cycloid profile limited by $13^\circ-119^\circ$ is favorable to the forces transmission. The beginning / ending of the cycloid profile correspond to some transmission angles unacceptable for the forces.

The usage of the entire length of the gearing line, from $\varphi=0^\circ ... 180^\circ$, is not rational, because, at the ending part of the gearing line, the pressure angle increase to $90^\circ$ showing us the existing possibilities of blocking.

We conclude that, from dynamic point of view, to transmit the forces is not rational to utilize the entire length of the cycloid profile

From point of view of the transmission of the forces, the favorable interval of the cycloid profile is limited by the tooth addendum circle with $r_s$ radius and the tooth dedendum circle with $r_t$ radius.