Comparison of Natural Frequencies Evaluated Experimentally and Numerically

Katarina Monkova¹, Peter Monka², Slavomir Hric³, Marek Urban⁴

Abstract: The article deals with the comparison of natural frequencies, which were obtained by two approaches: experimentally and numerically. As the components for natural frequencies evaluation were selected the simple beam and also the flange. Studying of natural frequencies within vibration of components or structures is important, because excessive vibrations due to resonance have been known to cause collapse of the structures. If designers had information regarding the natural frequencies of vibration of the structures under design, there would be a reduced possibility of resonance of the structures under the forces they are likely to be subjected to in their life spans. Experimental analyses were performed by means of modal hammer, laser vibrometer and analyser PULSE with its additional modules. Numerical approach to analysis was realized by means of Finite elements method within software PTC Creo.

Keywords: natural frequencies, experimental method, finite elements method, analysis

1 INTRODUCTION

Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate a mathematical model for its dynamic behaviour. The application of modal analysis is not limited to a specific engineering discipline. In recent years, extensive applications have been found not only in aerospace and acoustical engineering, but also in mechanical and automotive engineering. [1]

As the significance of dynamic behaviour of engineering structures is better appreciated, it becomes important to design them with proper consideration of dynamics. Finite element analysis, as a computer modelling approach, has provided engineers with a versatile design tool, especially when dynamic properties need to be perused. This numerical analysis requires rigorous theoretical guidance to ascertain meaningful outcomes in relation to structural dynamics. The principle of the numerical method is based on discretization of continuum to some number of finite elements, and these are being investigated parameters set out in the individual grid points. The theoretical basis of the technique is secured upon establishing the relationship between the vibration response at one location and excitation at the same or another location as a function of excitation frequency. [2,3,4]

For a harmonic force \( f(t) = F(\omega_0)e^{i\omega t} \), the response of the system is another harmonic function \( x(t) = X(\omega_0)e^{i\omega t} \), where \( F \) is a maximal force, \( X \) is a complex amplitude, \( \omega \) is an angular velocity and \( t \) is time. The ratio of the displacement response and the force input is often defined as the Frequency Response Function (FRF) of the system that is simply mathematically expressed by equation (1) [5]

\[
\alpha(\omega) = \frac{X(\omega)}{F(\omega)}
\]

The resultant FE model, which is in the form of mass and stiffness matrices, can be essential for further applications such as sensitivity analysis and prediction due to proposed structural changes. However, owing to the complexity and uncertainty of the structure, it is unrealistic to expect such an FE model to be faithfully representative. An essential approach is to take a measurement of the structure, derive its modal model and use it to correlate with the existing FE model in order to update it. The philosophy behind this model correlation is that the modal model derived from measurement, though incomplete due to lack of sufficient numbers of vibration modes and measured locations, truly represents the structure’s dynamic behaviour. Thus, it can be used to ‘correct’ the FE model, should any discrepancies occur between them. [6,7]

Experimental modal analysis is a system identification endeavour. The structure is a ‘black box’ that needs to be deciphered. The traditional approach is to provide the ‘black box’ with a known input, measure the output and proceed with the identification. Rapid improvement in measurement hardware and computing power in the last couple of decades has enabled us to make FRF measurement with multiple force inputs and multiple response outputs simultaneously. With multiple force inputs, it becomes possible to make the structure vibrate with reasonably uniform amplitudes rather than having great disparity of amplitudes across it under a single input. This type of measurement, if used properly, can result in more accurate FRF data and subsequently modal data. Time saving is accomplished. The demand on greater resources to conduct multiple input tests confines it to laboratories of sizable institutions. Numerical (finite element) and experimental modal analysis have become two pillars in structural dynamics. [8,9]

2 MODAL ANALYSIS OF A SIMPLE BEAM

A basic experiment for verification of two approaches to modal analysis was done. It was performed on the simple fixed beam with the rectangle profile made from the steel, see Fig. 1. The characteristic values of the beam were: \( l = 215 \text{ mm} \); \( b = 30 \text{ mm} \); \( h = 3,5 \text{ mm} \); \( E = 2.05.10^5 \) MPa; \( \rho = 7,85 \text{ kg/dm}^3 \), where \( l \), \( b \), \( h \) are dimensions of the beam, \( E \) is the modulus of elasticity and \( \rho \) is the material density.
2 MODAL ANALYSIS OF THE FLANGE PRODUCED BY ADDITIVE TECHNOLOGY

The aim of presented experiment was to identify the modal parameters of the flange as the component of planetary gear. Outer diameter of flange is $\phi 94.8$ mm and the diameter of concentric hole is $\phi 46$ mm. Material of flange is Maraging Steel 1.2709, because the part was produced by Direct Metal Laser Sintering technology. 3D model of investigated flange is presented in Fig 4.

The PULSE analyzer, model 2827 - 002 with its additional modules was plugged in for data processing. It consists of a measuring module type 3109 and communication module type 7533. As the vibration sensor, the piezoelectric accelerometer type 4374 was applied. Modal hammer Brüel & Kjær, type 8203 with plastic tip was used for part excitation. To amplify the sensor signal, the type of amplifier 2627–A was connected, which could also be used as a simple converter in default mode. In the experiment, two amplifiers of the same type were used, one for amplifying sensor signal and the second one for amplification of response to the alarm signal in modal hammer. The flange was studied while it was positioned on a soft foam. The complete measuring set is shown in Fig 5.

The measuring module was used to create the geometry model, and constructing the measuring points. The flange was excited in one point (point no. 1 in Fig. 6) and the response was measured in all 24 points.

The sampling frequency was setup at 6400 Hz with the accuracy 2 Hz. The final FRF is presented in the Fig. 7.

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**Fig. 1. The basic scheme of the fixed beam**

Experimental measures were done using the laser vibrometer Polytec PDV 100 with the system PULSE; the modal hammer Bruel & Kjaer 8206 was applied as the exciter and the data was processed by means the software MTC-Hammer. The beam was excited in one point (point no. 13) and the response was measured in all 24 points as it is shown in Fig. 2. [10]

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**Fig. 2. Experimental measuring with the specification of important points**

The sampling frequency was setup at 6400 Hz with the accuracy 2 Hz. The final FRF is presented in the Fig. 3. They are listed in Table 1, where the values of frequencies achieved numerically by means of FEM in software PTC Creo are also presented. Similarly, Table 2 shows natural shapes of the beam corresponding to deformations at natural frequencies obtained by both approaches.

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**Fig. 3. Final FRF function**

**Table 1. Acquired values of the natural frequencies**

<table>
<thead>
<tr>
<th>Method</th>
<th>$f_1$ [Hz]</th>
<th>$f_2$ [Hz]</th>
<th>$f_3$ [Hz]</th>
<th>$f_4$ [Hz]</th>
<th>$f_5$ [Hz]</th>
<th>$f_6$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical method (FEM)</td>
<td>62.1</td>
<td>388.8</td>
<td>849.9</td>
<td>1087.6</td>
<td>2128.7</td>
<td>2573.3</td>
</tr>
<tr>
<td>Experimental method (PULSE)</td>
<td>60</td>
<td>354</td>
<td>800</td>
<td>996</td>
<td>1956</td>
<td>2416</td>
</tr>
</tbody>
</table>

**Table 2. The first six mode shapes of the fixed beams**

<table>
<thead>
<tr>
<th>Method</th>
<th>1st mode shape</th>
<th>2nd mode shape</th>
<th>3rd mode shape</th>
<th>4th mode shape</th>
<th>5th mode shape</th>
<th>6th mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PULSE</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>7th mode shape</th>
<th>8th mode shape</th>
<th>9th mode shape</th>
<th>10th mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PULSE</td>
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**Fig. 4. 3D model of investigated flange**

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**Fig. 5. Measuring set**

**Fig. 6. The flange geometry with measured points**

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**Fig. 7. The final FRF function**
Within numerical method, the virtual model of the flange was prepared in software PTC Creo. Material properties include the specific density of MS1 steel 8 g/cm³ and Young's modulus is 172 GPA. [11] Finite elements grid has been defined with maximum edge of element 5 mm. In relation to the fact that the soft foam (on which the flange was positioned during experimental analysis) simulates unconstrained body, the same conditions were setup at numerical analysis.

Natural frequencies obtained by numerical and experimental approach are listed in the Table 1. Moreover, the first five natural shapes that respond to the natural frequencies were evaluated within the both methods. They are shown in the Table 2.

Table 3. Acquired values of the natural frequencies

<table>
<thead>
<tr>
<th>Method</th>
<th>(f_1) [Hz]</th>
<th>(f_2) [Hz]</th>
<th>(f_3) [Hz]</th>
<th>(f_4) [Hz]</th>
<th>(f_5) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical method (FEM)</td>
<td>2 466,29</td>
<td>2 438,98</td>
<td>5 142,06</td>
<td>5 722,82</td>
<td>8 598,87</td>
</tr>
<tr>
<td>Experimental method (PULSE)</td>
<td>2 265,22</td>
<td>2 451,48</td>
<td>5 109,27</td>
<td>5 683,64</td>
<td>8 566,67</td>
</tr>
</tbody>
</table>

3 CONCLUSIONS

In the past two decades, modal analysis has become a major technology in the quest for determining, improving and optimizing dynamic characteristics of engineering structures. Once the modal model of a structure is adequately determined, it becomes possible to predict structural response for any input forces that could be incisive in forecasting the fatigue life of the structure.

The article deals with numerical and experimental modal analysis of two components: simple beam and flange. Within the research, the experimental and computational models were created due to the verification of obtained modal parameters. Numerical analysis was carried out by means of FEM method using PTC Creo software. The results obtained by experimental modal analysis were evaluated by measuring system PULSE. Experimental and numerical measurements have shown that the values of natural frequencies along with natural shapes are comparable. The differences between the obtained values were caused by type of constraints and also by a different number of finite elements used at both methods. Obtained data of modal analysis will be the base for the dynamic analysis and for the next experiments that authors are going to perform.

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REFERENCES

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